

NASA Technical Memorandum 100671

**ANALYTICAL MODEL OF A FIVE DEGREE
OF FREEDOM MAGNETIC SUSPENSION AND
POSITIONING SYSTEM**

(NASA-TM-100671) ANALYTICAL MODEL OF A FIVE
DEGREE OF FREEDOM MAGNETIC SUSPENSION AND
POSITIONING SYSTEM (NASA) 18 P CSCL 13H

N89-21136

G3/31 Unclass
0201512

Nelson J. Groom

March 1989



National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665-5225

SUMMARY

An analytical model of a five degree of freedom magnetic suspension and positioning system is presented. The suspended element is a cylinder which is composed of permanent magnet material and the magnetic actuators are air core electromagnets mounted in a planar array. The analytical model consists of an open loop representation of the suspension and positioning system with electromagnet currents as inputs and displacements and rates in inertial coordinates as outputs. The uncontrolled degree of freedom is rotation about the long axis of the suspended cylinder.

INTRODUCTION

This paper describes an analytical model of a five degree of freedom magnetic suspension and positioning system. The suspended element is a cylinder which is composed of permanent magnet material and the magnetic actuators are air core electromagnets mounted in a planar array. The uncontrolled degree of freedom is rotation about the long axis of the suspended cylinder. The electromagnet array is mounted horizontally with the suspended element suspended above the array by repulsive forces. The analytical model consists of an open loop representation of the suspension and positioning system with electromagnet currents as inputs and displacements and rates in inertial coordinates as outputs.

EQUATIONS OF MOTION

This section presents the equations of motion of the suspended element in general terms. The suspended element, or core, is a cylinder which is composed of permanent magnet material. The core is suspended, by repulsive forces, over a planar array of five electromagnets mounted in a circular arrangement. Since only five degrees of freedom are being controlled, this represents the minimum number of actuators. The permanent magnet material provides the necessary magnetic field which interacts with the fields of the suspension electromagnets to produce suspension and positioning forces and torques. A schematic representation of this system which shows the coordinate systems and initial alignment is shown in figure 1. A set of orthogonal \bar{x} , \bar{y} , and \bar{z} body fixed axes defines the motion of the core with respect to inertial space. The core axis system is initially aligned with an orthogonal x , y , and z system fixed in inertial space. A set of orthogonal x_b , y_b , and z_b axes, also fixed in inertial space, define the location of the electromagnet array with respect to the x , y , and z system. The x and y axes are parallel to the x_b and y_b axes respectively, and the z and z_b axes are coincident. The centers of the two axis systems are separated by the distance h .

The development of the core equations of motion follow basically the development of the spacecraft equations of motion presented in reference 1. The assumptions used for deriving the equations of motion are: (1) the core is a rigid body, (2) the core has negligible products of inertia, and (3) there is no motion about the \bar{x} axis. Referring to figure 1, the \bar{x} axis is an axis of symmetry so that

$$I_{\bar{y}} = I_{\bar{z}} = I_c \quad (1)$$

The momentum of the core in core coordinates can be written as

$$\{\bar{H}\} = [I]\{\dot{\bar{\Omega}}\} \quad (2)$$

where

$$[I] = \begin{bmatrix} \bar{I}_{\bar{x}} & 0 & 0 \\ 0 & I_c & 0 \\ 0 & 0 & I_c \end{bmatrix} \quad (3)$$

and

$$\{\bar{\Omega}\} = [0 \ \Omega_{\bar{y}} \ \Omega_{\bar{z}}] \quad (4)$$

A bar over a variable indicates that it is referenced to core coordinates. The total torque, $\{\bar{T}\}$, about the center of mass of the core referenced to core coordinates is

$$\{\bar{T}\} = \{\dot{\bar{H}}\} = [I]\{\dot{\bar{\Omega}}\} + \{\bar{\Omega}\} \times ([I]\{\bar{\Omega}\}) \quad (5)$$

which can be written as

$$\{\bar{T}\} = [I]\{\dot{\bar{\Omega}}\} + [\bar{\Omega}][I]\{\bar{\Omega}\} \quad (6)$$

where $[\bar{\Omega}]$ is the skew symmetric cross product matrix defined by

$$[\bar{\Omega}] = \begin{bmatrix} 0 & -\Omega_{\bar{z}} & \Omega_{\bar{y}} \\ \Omega_{\bar{z}} & 0 & -\Omega_{\bar{x}} \\ -\Omega_{\bar{y}} & \Omega_{\bar{x}} & 0 \end{bmatrix} \quad (7)$$

$\{\bar{T}\}$ can be written

$$\{\bar{T}\} = \{\bar{T}_m\} + \{\bar{T}_d\} \quad (8)$$

where $\{\bar{T}_m\}$ are the control torques on the core produced by the electromagnets and $\{\bar{T}_d\}$ are external disturbance torques. Rearranging terms in equation (6) results in

$$\{\dot{\bar{\Omega}}\} = [I]^{-1}(\{\bar{T}\} - [\bar{\Omega}][I]\{\bar{\Omega}\}) \quad (9)$$

From equation (1) and the assumption that $\Omega_{\bar{x}} = 0$, the cross product term becomes zero and equation (9) reduces to

$$\{\dot{\bar{\Omega}}\} = (1/I_c)\{\bar{T}\} \quad (10)$$

The angular rates of the core are obtained by integrating equation (10). The core Euler rates can be written as

$$\{\dot{\theta}\} = [T_E]\{\bar{\Omega}\} \quad (11)$$

where

$$[T_E] = \begin{bmatrix} 1 & \tan \theta_y \sin \theta_x & \tan \theta_y \cos \theta_x \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sec \theta_y \sin \theta_x & \sec \theta_y \cos \theta_x \end{bmatrix} \quad (12)$$

gives the transformation matrix from core body rates to core Euler rates for a 3, 2, 1 (z, y, x) rotation sequence.

The forces on the core, in core coordinates, can be expressed as

$$\{\bar{F}\} = m_c \{ \{\dot{\bar{V}}\} + [\bar{\Omega}]\{\bar{V}\} \} \quad (13)$$

where m_c is the mass of the core, $[\bar{\Omega}]$ is defined above, and $\{\bar{V}\}$ is core velocity. $\{\bar{F}\}$ can be written

$$\{\bar{F}\} = \{\bar{F}_m\} + \{\bar{F}_d\} \quad (14)$$

where $\{\bar{F}_m\}$ are the control forces on the core produced by the electromagnets and $\{\bar{F}_d\}$ are disturbance forces. Rearranging (13) results in

$$\{\dot{\bar{V}}\} = (1/m_c)\{\bar{F}\} - [\bar{\Omega}]\{\bar{V}\} \quad (15)$$

The core translational rates are obtained by integrating (15). The core translational rates in inertial coordinates, $\{V\}$, become

$$\{V\} = [T_m]^{-1}\{\bar{V}\} \quad (16)$$

where

$$[T_m]^{-1} = \begin{bmatrix} c \theta_z c \theta_y & (c \theta_z s \theta_y s \theta_x - s \theta_z c \theta_x) & (c \theta_z s \theta_y c \theta_x + s \theta_z s \theta_x) \\ s \theta_z c \theta_y & (s \theta_z s \theta_y s \theta_x + c \theta_z c \theta_x) & (s \theta_z s \theta_y c \theta_x - c \theta_z s \theta_x) \\ -s \theta_y & c \theta_y s \theta_x & c \theta_y c \theta_x \end{bmatrix} \quad (17)$$

gives the vector transformation from core coordinates to inertial coordinates. Because of the size of the expression, sin has been shortened to s and cos has been shortened to c. The displacement of the core center of mass in inertial coordinates is obtained by integrating equation (16). Since the core is to be actively controlled, the assumption is made that the rates will be small and that products of rates can be neglected. Under this assumption, the cross product term in equation (15), $\{\dot{\Omega}\}\{\tilde{V}\}$, can be dropped and the result is

$$\{\dot{\tilde{V}}\} = (1/m_c)\{\tilde{F}\} \quad (18)$$

MAGNETIC TORQUES AND FORCES

The equations for torques and forces on a magnetic core which are produced by air core electromagnets have been presented and discussed in a number of papers on wind tunnel magnetic suspension. For example, see references 2-5. This section presents the pertinent equations in general terms.

The torque on a magnetic core, in a given orthogonal x, y, z coordinate system, in a nonuniform, but steady, magnetic field can be written as

$$\{T\} = \int (d\{M\} \times \{B\}) dV \quad (19)$$

where the integral is taken over the volume of the core and M is the magnetization of the core in amp/meter, B is flux density in Tesla, V is volume in cubic meters, and torque, T, is in Newton-meters. The force on the core can be written as

$$\{F\} = \int (d\{M\} \cdot \nabla) \{B\} dV \quad (20)$$

where the force, F, is in Newtons and ∇ is the gradient operator defined as

$$\nabla^T = [\partial/\partial x \quad \partial/\partial y \quad \partial/\partial z] \quad (21)$$

Since the size of the core is small relative to the size of the electromagnets and the gaps involved, a reasonable approximation of the torques and forces which are produced can be obtained by making the assumption that the field and gradient

components are uniform over the volume of the core. Under this assumption, the torque becomes

$$\{T\} = (Vol)(\{M\} \times \{B\}) \quad (22)$$

where Vol is the volume of the core and $\{B\}$ is calculated at the center of the core. Equation (22) can be written as

$$\{T\} = (Vol)[M]\{B\} \quad (23)$$

where $[M]$ is the cross product matrix. The force becomes

$$\{F\} = (Vol)(\{M\} \cdot \nabla)\{B\} \quad (24)$$

Taking the dot product, $\{M\} \cdot \nabla$, results in the scalar

$$\{M\} \cdot \nabla = \{M\}^T \nabla = (M_x \partial/\partial x + M_y \partial/\partial y + M_z \partial/\partial z) \quad (25)$$

Equation (24) then becomes

$$\{F\} = (Vol) \begin{bmatrix} M_x \partial B_x / \partial x + M_y \partial B_x / \partial y + M_z \partial B_x / \partial z \\ M_x \partial B_y / \partial x + M_y \partial B_y / \partial y + M_z \partial B_y / \partial z \\ M_x \partial B_z / \partial x + M_y \partial B_z / \partial y + M_z \partial B_z / \partial z \end{bmatrix} \quad (26)$$

The notation can be simplified considerably by letting B_{ij} represent $\partial B_i / \partial j$. By factoring M out as a vector, (26) can be written in the form

$$\{F\} = (Vol)[\partial B]\{M\} \quad (27)$$

where

$$[\partial B] = \begin{bmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{bmatrix} \quad (28)$$

in simplified notation. From Maxwell's equations, in the region of the core $\nabla \times B = 0$ which results in

$$B_{xy} = B_{yx} \quad (29)$$

$$B_{xz} = B_{zx} \quad (30)$$

$$B_{yz} = B_{zy} \quad (31)$$

Also $\nabla \cdot B = 0$ which results in

$$B_{xx} + B_{yy} + B_{zz} = 0 \quad (32)$$

Equations (23) and (27) are written in terms of a given orthogonal coordinate system. In the magnetic suspension and positioning system $\{\tilde{M}\}$ is defined in core coordinates and $\{B\}$ is defined in inertial coordinates. Therefore, in order to calculate torque in core coordinates, $\{B\}$ has to be transformed into the same coordinate system. This results in

$$\{\tilde{T}\} = (Vol)[\tilde{M}][T_m]\{B\} \quad (33)$$

In order to perform the operations involved in the force calculations, $\{\tilde{M}\}$ is transformed into inertial coordinates

$$\{M\} = [T_m]^{-1}\{\tilde{M}\} \quad (34)$$

and the resulting force transformed back into core coordinates

$$\{\tilde{F}\} = (Vol)[T_m][\partial B][T_m]^{-1}\{\tilde{M}\} \quad (35)$$

Also, since the core is made of permanent magnet material which is magnetized along the x axis, M can be written as

$$\{\tilde{M}\} = [M_x \ 0 \ 0] \quad (36)$$

SYSTEM EQUATIONS

In this section the magnetic torques and forces are combined with the equations of motion of the permanent magnet core to produce an open loop model of the magnetic suspension and positioning system. Substituting (33) into (10) results in

$$\{\dot{\tilde{n}}\} = (1/I_c) \{Vol([\tilde{M}][T_m]\{B\}) + \{\tilde{T}_d\}\} \quad (37)$$

Substituting (35) into (18) results in

$$\{\dot{\tilde{v}}\} = (1/m_c) \{Vol([T_m][\partial B][T_m]^{-1}\{\tilde{M}\}) + \tilde{F}_d\} \quad (38)$$

This gives the equations of motion in core coordinates in terms of $\{B\}$ and $[\partial B]$, the gradients of $\{B\}$. Since the core is assumed to be accurately held to a fixed point in translation with only small displacements being allowed about the operating point, $\{B\}$ and $[\partial B]$ at this point are assumed to be essentially constant. This means that $\{B\}$ and $[\partial B]$ can be represented as functions of coil currents only. If $\{B\}$ and $[\partial B]$ are calculated at the operating point, in inertial coordinates, for the maximum current in each coil, then $\{B\}$ can be written as

$$\{B\} = (1/I_{\max}) [K_B] \{I\} \quad (39)$$

where I_{\max} is the maximum coil current, $[K_B]$ is a 3×5 matrix whose elements represent the values of $\{B\}$ produced by a corresponding coil driven by the maximum current and

$$\{I\}^T = [I_1 \ I_2 \ I_3 \ I_4 \ I_5] \quad (40)$$

The gradients can be put in the same form by arranging the elements of $[\partial B]$ as a column vector. This results in

$$\{\partial B\} = (1/I_{\max}) [K_{\partial B}] \{I\} \quad (41)$$

where $\{\partial B\}$ is a nine element column vector containing the gradients of $\{B\}$, and $[K_{\partial B}]$ is a 9×5 matrix whose elements represent the values of $\{\partial B\}$ produced by a corresponding coil driven by the maximum current. Each element of $\{\partial B\}$, for example B_{xx} , can be written in the form

$$B_{xx} = (1/I_{\max}) [K_{xx}] \{I\} \quad (42)$$

where $[K_{xx}]$ is a 1×5 matrix containing values of B_{xx} produced by a corresponding coil driven by the maximum current. An analytical model of the magnetic suspension and positioning system can now be put into block diagram form as shown in figure 2. This model is nonlinear because of the combination of states resulting from the coordinate transformations and is of the form

$$\dot{x} = f(x, u) \quad (43)$$

where x is given by

$$x^T = [\bar{\Omega}_y \ \bar{\Omega}_z \ \theta_y \ \theta_z \ \bar{V}_x \ \bar{V}_y \ \bar{V}_z \ x \ y \ z] \quad (44)$$

and the input u is given by

$$u^T = [I_1 \ I_2 \ I_3 \ I_4 \ I_5] \quad (45)$$

The system can be linearized about a nominal operating point x_0, u_0 by performing a Taylor series expansion (ref. 6). Ignoring second order terms, the Taylor series expansion about x_0, u_0 gives

$$\begin{aligned} \dot{x}_0 + \delta \dot{x} &= f(x_0 + \delta x, u_0 + \delta u) \\ &= f(x_0, u_0) + \left. \frac{\partial f(x, u)}{\partial x} \right|_{x_0, u_0} \delta x + \left. \frac{\partial f(x, u)}{\partial u} \right|_{x_0, u_0} \delta u \end{aligned} \quad (46)$$

The equations for computing u_0 for a given x_0 are presented in the appendix. Subtracting out \dot{x}_0 results in

$$\delta \dot{x} = \bar{A} \delta x + \bar{B} \delta u \quad (47)$$

where $\bar{A} = \left. \frac{\partial f(x, u)}{\partial x} \right|_{x_0, u_0}$ and $\bar{B} = \left. \frac{\partial f(x, u)}{\partial u} \right|_{x_0, u_0}$. These equations can be used to calculate feedback gains to stabilize and control the core about the selected operating point. Figure 3 illustrates this approach in block diagram form.

CONCLUDING REMARKS

This paper has developed an analytical model of a five degree of freedom magnetic suspension and positioning system. The system consists of a suspended element which is a cylinder composed of permanent magnet material and magnetic actuators which are air core electromagnets mounted in a planar array. The analytical model is an open loop representation with displacements and rates in inertial coordinates as outputs and electromagnet currents as inputs. The number of electromagnets used in the model is the minimum number of five. However, adding electromagnets should be straightforward. The model should be useful in analyses and simulations in the development of control systems approaches and in evaluations of overall systems performance.

REFERENCES

1. Groom, Nelson J.: Fixed-Base and Two-Body Equations of Motion for an Annular Momentum Control Device (AMCD). NASA TM-78644, March 1978.
2. Basmajian, V. V.; Copeland, A. B.; and Stephens, T.: Studies Related to the Design of a Magnetic Suspension and Balance System. NASA CR-66233, February 1966.
3. Stephens, Timothy: Design, Construction, and Evaluation of a Magnetic Suspension and Balance System for Wind Tunnels. NASA CR-66903, November 1969.
4. Covert, Eugene E.; Finston, Morton; Vlajinac, Milan; and Stephens, Timothy: Magnetic Balance and Suspension Systems for Use in Wind Tunnels. Progress in Aerospace Sciences, Vol. 14. Pergamon Press, 1973.
5. Britcher, Colin P.: Some Aspects of Wind Tunnel Magnetic Suspension Systems With Special Application at Large Scales. NASA CR-172154, September 1983.
6. Brogan, William L.: Modern Control Theory. Second ed. Prentice-Hall, Inc., 1985.

APPENDIX

INITIAL CONDITIONS

This appendix develops a linearized model of the Magnetic Suspension And Positioning System shown in figure 2 by assuming that the suspended core is held fixed at a given initial operating point. The linearized model is then used to calculate initial conditions on the inputs. The assumptions made in developing the model are that the suspended core is held fixed at an initial nominal operating point which can be anywhere within 360 degrees in yaw (θ_z) but will be zero in pitch (θ_y). Also, the only nonzero disturbance input is assumed to be the weight of the core. Under the above assumptions, equation (17) becomes

$$[T_m]^{-1} = \begin{bmatrix} c\theta_z & -s\theta_z & 0 \\ s\theta_z & c\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (A1)$$

and

$$[T_m] = \begin{bmatrix} c\theta_z & s\theta_z & 0 \\ -s\theta_z & c\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (A2)$$

where θ_z is a given fixed yaw angle. From equation (33) the torque in core coordinates is

$$\{\dot{T}\} = (Vol)[\dot{M}][T_m]\{B\} \quad (A3)$$

Substituting (A2) into (A3) and carrying out the matrix operations results in

$$\{\dot{T}\} = (Vol) \begin{bmatrix} 0 \\ -M_x B_z \\ M_x (-s\theta_z B_x + c\theta_z B_y) \end{bmatrix} \quad (A4)$$

Factoring out M_x and substituting from equation (38) for B gives

$$\{\dot{T}\} = (Vol)M_x/I_{max} \begin{bmatrix} 0 \\ -[K_z] \\ -s\theta_z [K]_x + c\theta_z [K]_y \end{bmatrix} \{I\} \quad (A5)$$

Equation (A5) can be written in the form

$$\{\bar{T}\} = (Vol)M_x/I_{max}[KT]\{I\} \quad (A6)$$

where $[KT]$ is defined as

$$[KT] = \begin{bmatrix} 0 \\ -[K_z] \\ -s\theta_z[K_x] + c\theta_z[K_y] \end{bmatrix} \quad (A7)$$

Substituting (A1) and (A2) into (35) and performing the matrix operations results in

$$\{\bar{F}\} = (Vol)M_x \begin{bmatrix} c^2\theta_z B_{xx} + c\theta_z s\theta_z B_{xy} + c\theta_z s\theta_z B_{xy} + s^2\theta_z B_{yy} \\ -c\theta_z s\theta_z B_{xx} - s^2\theta_z B_{xy} + c^2\theta_z B_{xy} + c\theta_z s\theta_z B_{yy} \\ c\theta_z B_{xz} + s\theta_z B_{yz} \end{bmatrix} \quad (A8)$$

where the relation $B_{xy} = B_{yx}$ has been used. From equation (40), equation (A8) can be written as

$$\{\bar{F}\} = (Vol)M_x/I_{max} \begin{bmatrix} c^2\theta_z[K_{xx}] + 2c\theta_z s\theta_z[K_{xy}] + s^2\theta_z[K_{yy}] \\ -c\theta_z s\theta_z[K_{xx}] + (c^2\theta_z - s^2\theta_z)[K_{xy}] + c\theta_z s\theta_z[K_{yy}] \\ c\theta_z[K_{xz}] + s\theta_z[K_{yz}] \end{bmatrix} \{I\} \quad (A9)$$

Equation (A9) can be written in the form

$$\{\bar{F}\} = (Vol)M_x/I_{max}[KF]\{I\} \quad (A10)$$

where $[KF]$ is defined as

$$[KF] = \begin{bmatrix} c^2\theta_z[K_{xx}] + 2c\theta_z s\theta_z[K_{xy}] + s^2\theta_z[K_{yy}] \\ -c\theta_z s\theta_z[K_{xx}] + (c^2\theta_z - s^2\theta_z)[K_{xy}] + c\theta_z s\theta_z[K_{yy}] \\ c\theta_z[K_{xz}] + s\theta_z[K_{yz}] \end{bmatrix} \quad (A11)$$

Dropping the zero term, which is T_x , from (A6) and combining with (A10) results in

$$\begin{Bmatrix} \bar{T} \\ \bar{F} \end{Bmatrix} = (Vol)M_x/I_{max} \begin{bmatrix} [KT] \\ [KF] \end{bmatrix} \{I\} \quad (A12)$$

where $\begin{Bmatrix} \bar{T} \\ \bar{F} \end{Bmatrix}$ is a 5x1 column vector and $\begin{bmatrix} [KT] \\ [KF] \end{bmatrix}$ is a 5x5 matrix.

The solution for the initial currents is then

$$\{I\} = I_{max}/(Vol)M_x \begin{bmatrix} [KT] \\ [KF] \end{bmatrix}^{-1} \begin{Bmatrix} \bar{T} \\ \bar{F} \end{Bmatrix} \quad (A13)$$

As mentioned earlier, the only nonzero element of $\begin{Bmatrix} \bar{T} \\ \bar{F} \end{Bmatrix}$ is the weight of the core.

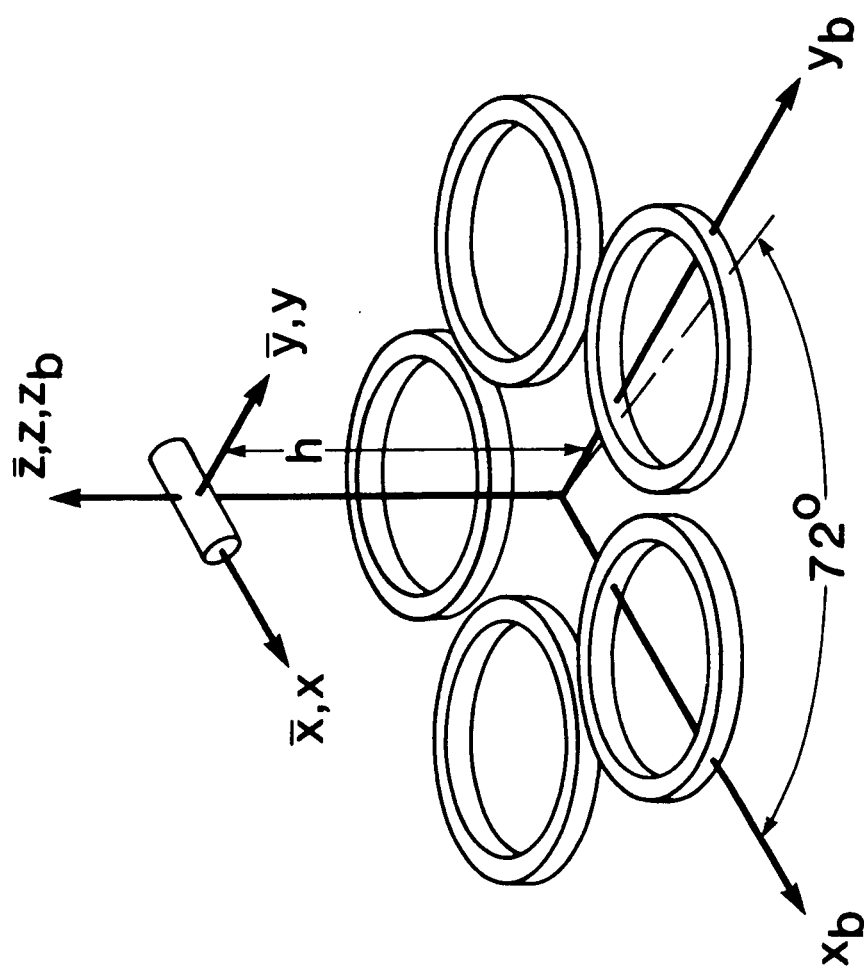


Figure 1.-Initial coordinate system alignment for magnetic suspension & positioning system.

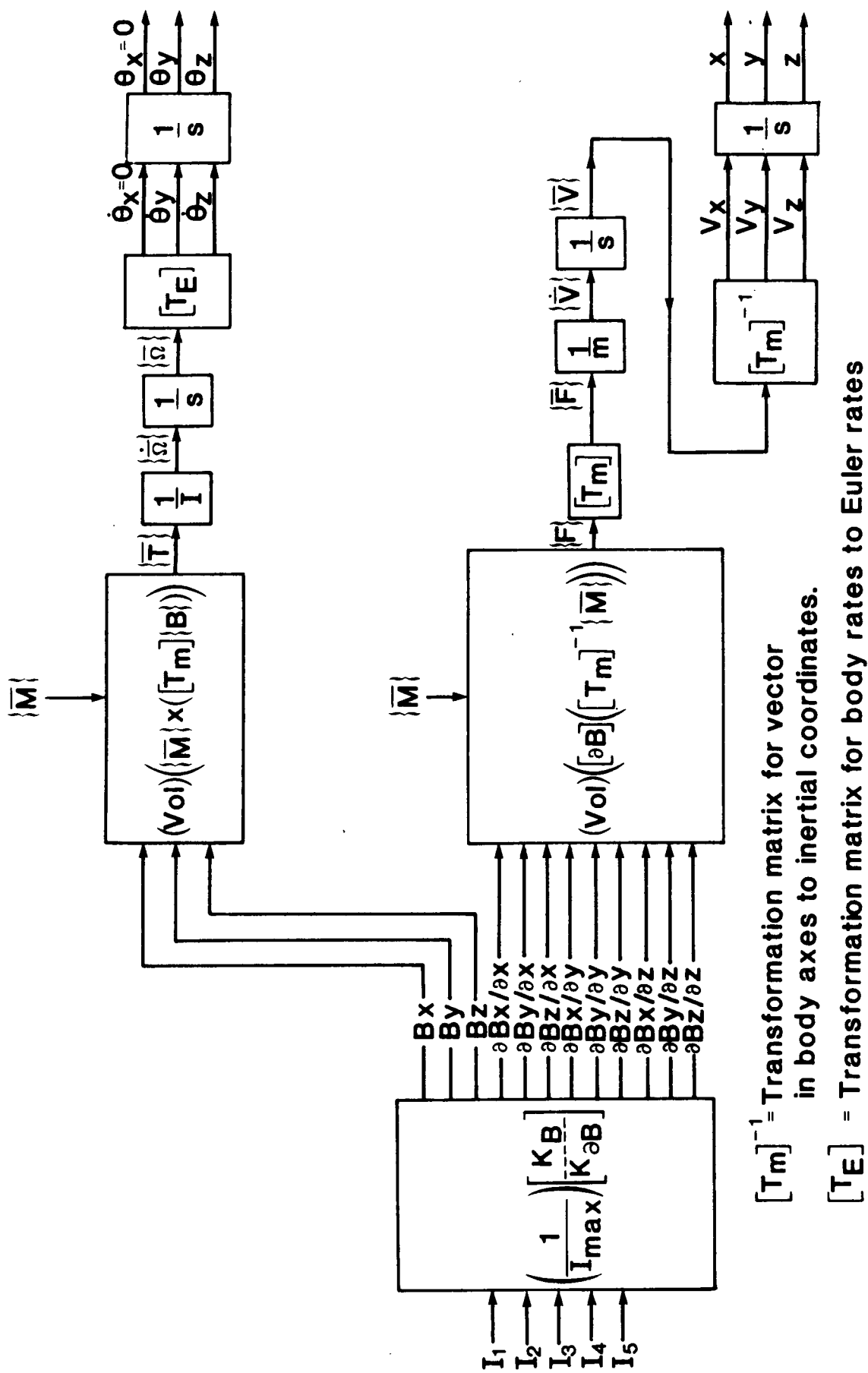
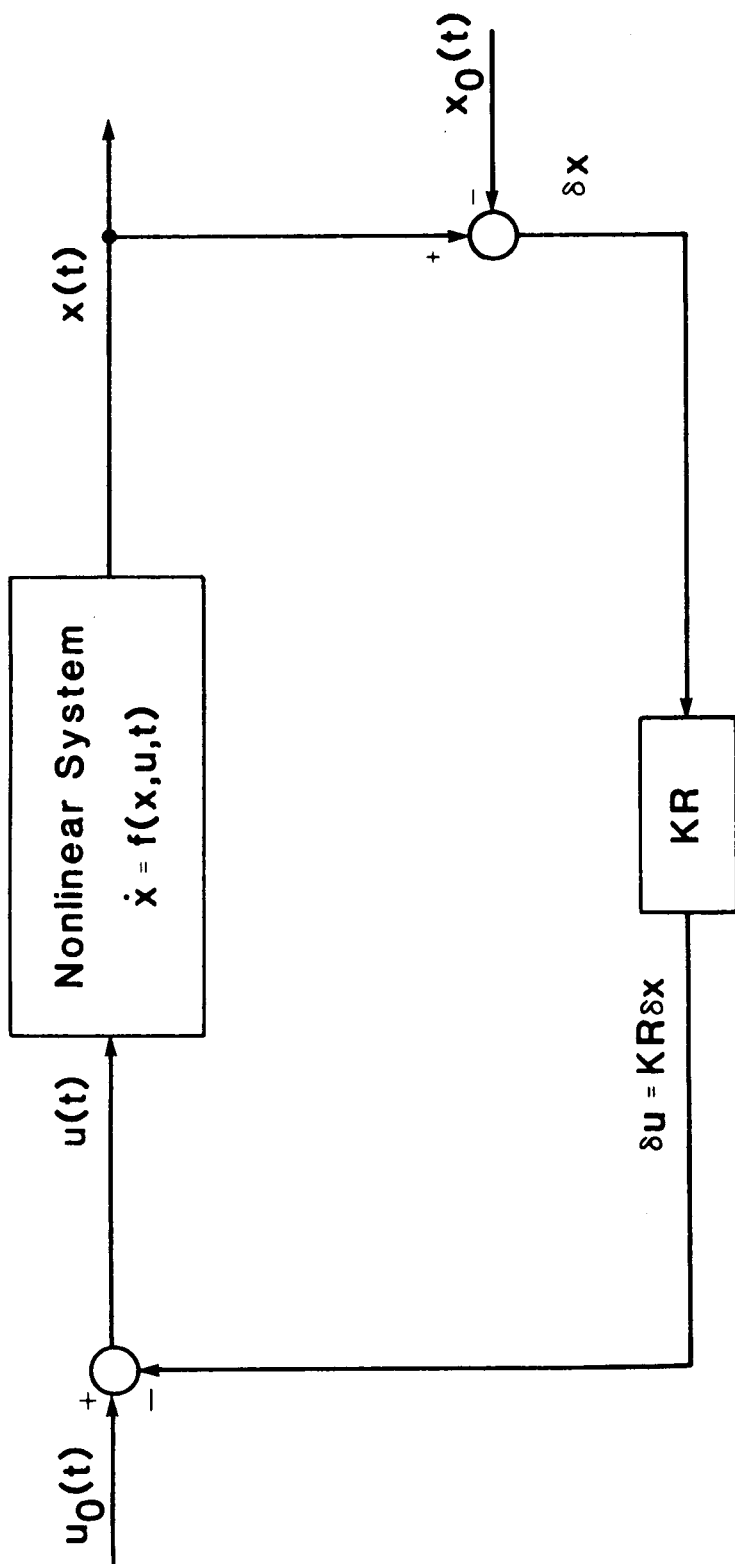


Figure 2.-Block diagram of analytical model of magnetic suspension & positioning system.



Linear System: $\delta \dot{x} = \bar{A} \delta x + \bar{B} \delta u$

Figure 3.-Approach for controlling nonlinear system about nominal operating point x_0, u_0 .



Report Documentation Page

1. Report No. NASA TM-100671	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle Analytical Model of a Five Degree of Freedom Magnetic Suspension and Positioning System		5. Report Date March 1989	
		6. Performing Organization Code	
7. Author(s) Nelson J. Groom		8. Performing Organization Report No.	
		10. Work Unit No. 585-01-61-08	
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665-5225		11. Contract or Grant No.	
		13. Type of Report and Period Covered Technical Memorandum	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546-0001		14. Sponsoring Agency Code	
15. Supplementary Notes			
16. Abstract An analytical model of a five degree of freedom magnetic suspension and positioning system is presented. The suspended element is a cylinder which is composed of permanent magnet material and the magnetic actuators are air core electromagnets mounted in a planar array. The analytical model consists of an open loop representation of the suspension and positioning system with electromagnet currents as inputs and displacements and rates in inertial coordinates as outputs. The uncontrolled degree of freedom is rotation about the long axis of the suspended cylinder.			
17. Key Words (Suggested by Author(s)) Magnetic Suspension Magnetic Levitation Pointing System Magnetic Suspension Model		18. Distribution Statement Unclassified-Unlimited Subject Category 31	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 16	22. Price A03